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Seat No. _______________

HAJ-003-1015025

B. Sc. (Sem. V) (CBCS) Examination

May - 2023

Physics - 501

(Mathematical Physics, Classical Mechanics & Quantum Mechanics) (Old Course)

Faculty Code : 003

Subject Code : 1015025

Time : $2\frac{1}{2}$ / Total Marks : **70**

Instructions : (1) All questions are compulsory.

(2) Symbols have their usual meanings.

(3) Figures to the right indicate marks.

Physical constants :

$h = 6.62 \times 10^{-34}$ Js, $\hbar = 1.055 \times 10^{-34}$ Js, Mass of an electron = 9.1×10^{-31} kg

1 (a) Answer the following objective questions: **4**

(1) Write the value of b_n for an odd function (sine series).

(2) Sin x is an odd function. True or false?

(3) Write the Fourier equation.

(4) Write the complex form of Fourier series.

(b) Answer any one question : **2**

(1) Expand $f(x) = x, \ -\pi < x < \pi$.

(2) Expand

$$f(x) = 0, \ -\pi < x < 0$$
$$= 1, \ 0 < x < \pi$$

(c) Answer any one question : **3**

(1) Explain the action of a full wave rectifier based on Fourier analysis.

(2) Explain square wave based on Fourier analysis.

(d) Answer any one question in detail : **5**

(1) What is Fourier series? Derive Fourier coefficients.

(2) Explain Fourier transform, Fourier sine transform and Fourier cosine transform.

2 (a) Answer the following objective questions : **4**

(1) Define generalized displacement.

(2) Name the two types of constraints classified on the basis of velocity.

(3) What is meant by degree of freedom?

(4) Write Euler-Lagrange differential equation.

(b) Answer any one question : **2**

(1) The kinetic energy and potential energy of a simple harmonic oscillator are respectively $T = \frac{1}{2} m\dot{y}^2$ and $V = \frac{1}{2} m\omega^2 y^2$. Find Lagrange's equation of motion.

(2) For a compound pendulum, kinetic energy $T = \frac{1}{2} l\dot{\theta}^2$ and potential energy V = –mghcosθ. Find the Lagrange's equation of motion.

(c) Answer any one question : **3**

(1) Explain virtual work.

(2) Obtain Lagrange's equation for a simple pendulum.

(d) Answer any one question in detail : **5**

(1) Derive Lagrange's equation from D'Alembert's principle.

(2) Derive Hamilton's principle using D'Alembert's principle.

3 (a) Answer the following objective questions : **4**

(1) If $\frac{\partial L}{\partial q_j} = 0$, then q_j can be defined as....

(2) What is phase space?

(3) Write the Hamiltonian equation for a charged particle in an electromagnetic field.

(4) What are the Hamilton's canonical equations of motion?

(b) Answer any one question : **2**

(1) Obtain Hamilton's equations for a system whose Lagrangian is given as,

$$L = \frac{1}{2} m\left(\dot{y}^2 + l^2\dot{\theta}^2 + 2\dot{y}l\dot{\theta}\cos\theta\right) - mgl\left(1 - \cos\theta\right).$$

(2) Find the Hamiltonian for the Lagrangian

$$L\left(x, \dot{x}\right) = \frac{\dot{x}^2}{2} - \frac{\omega^2 x^2}{2} - \alpha x^3 + \beta x\dot{x}^2.$$

(c) Answer any one question : **3**

(1) Explain the physical significance of H.

(2) Obtain the Hamilton's equation for a linear harmonic oscillator.

(d) Answer any one question in detail : **5**

(1) Explain generalized velocity and generalized momentum.

(2) Obtain the Hamilton's canonical equations of motion.

4 (a) Answer the following objective questions : **4**

(1) The expectation value of momentum is defined as

$< p >= \int \psi^* \left(r, t\right) \left(-i\hbar\nabla\right) \psi\left(r, t\right) d\tau.$ True or false?

(2) For a normalized wavefunction $\int_{-\infty}^{\infty} |\psi|^2 \, d\tau = ...$

(3) What is photoelectric effect ?

(4) The operator correspondence of energy is $-i\hbar\nabla$. True or false?

(b) Answer any one question : **2**

(1) Normalize the wavefunction $\psi\left(x\right) = Ae^{ikx}$ over the region $-a<x<a$.

(2) Calculate the uncertainty in the measurement of position if the uncertainty in the measurement of momentum is $6.0x\ 10^{-18}\ kgms^{-1}$.

(c) Answer any one question : **3**

(1) Explain box normalization with an example.

(2) Derive time independent Schrodinger equation.

(d) Answer any one question in detail : **5**

(1) Derive the one dimensional Schrodinger equation and extend it to three dimensions.

(2) Explain Photoelectric effect in detail.

5 (a) Answer the following objective questions : **4**

(1) What is a self adjoint operator?

(2) $[x, p_x]$ = ...

(3) The equation $\frac{d^2u}{dy^2} - 2y\frac{du}{dy} + (\lambda - 1)u = 0$ is known as _______ differential equation.

(4) $(A + B)^{\dagger}$ = ...

(b) Answer any one question : **2**

(1) Prove that $[y, p_x] = 0$.

(2) If $H = \frac{P^2}{2m} + \frac{1}{2}m\omega^2 x^2$ then, prove that $[x, H] = \frac{i\hbar P}{m}$.

(c) Answer any one question : **3**

(1) Prove that $[L_x, L_y] = i\hbar L_z$

(2) Obtain the Schrodinger equation for a harmonic oscillator (wave equation for an oscillator).

(d) Answer any one question in detail. **5**

(1) Explain angular momentum operator. Derive the expressions for L_x, L_y and L_z and hence obtain the relation

$$L^2 = -\hbar^2\left[\frac{1}{\sin\theta}\frac{\partial}{\partial\theta}\left(\sin\theta\frac{\partial}{\partial\theta}\right) + \frac{1}{\sin^2\theta}\frac{\partial^2}{\partial\phi^2}\right].$$

(2) Write Hermite's differential equation and find its solutions.